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Finite Element Analysis of Helical Gear Using Three-Dimensional Cad Model

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Abstract

Gears are one of the most critical components in mechanical power transmission systems. The bending and surface strength of the gear tooth are considered to be one of the main contributors for the failure of the gear in a gear set. Thus, analysis of stresses has become popular as an area of research on gears to minimize or to reduce the failures and for optimal design of gears. This paper investigates finite element model for monitoring the stresses induced of tooth flank, tooth fillet during meshing of gears. The involute profile of helical gear has been modeled and the simulation is carried out for the bending and contact stresses and the same have been estimated. To estimate bending and contact stresses, 3D models are generated by modeling software CATIA V5 and simulation is done by finite element software package ANSYS 14.0. Analytical method of calculating gear bending stresses uses Lewis and AGMA bending equation. For contact stresses Hertz and AGMA contact equation are used. Study is conducted by varying the face width to find its effect on the bending stress of helical gear. It is therefore observed that the maximum bending stress decreases with increasing face width. The stresses found from ANSYS results are compared with those from theoretical and AGMA values.

Keywords: Helical Gear, Stress Calculation, Bending Stress, Contact Stress, Finite Element Analysis.

Introduction

Gears are used to transmit power and motion from one shaft to another. Helical gears are currently being used increasingly as a power transmitting gear owing to their relatively smooth and silent operation, large load carrying capacity and higher operating speed. Helical gears have a smoother operation than the spur gears because of a large helix angle that increases the length of the contact lines. Designing highly loaded helical gears for power transmission systems that are good in strength and low level in noise necessitate suitable analysis methods that can easily be put into practice and also give useful information on contact and bending stresses [1]. Gears are used to change the speed, magnitude, and direction of a power source. Gears are being most widely used as the mechanical elements of power transmission. When two gears with unequal numbers of teeth are combined, a productive output is realized with both the angular speeds and the torques of the two gears differing through a simple relationship. AGMA [1] and ISO [2] standards generally are being used as the strength standard for the design of spur, helical, and worm gears. The strength determined from the AGMA and ISO standards is valid under the

assumption that the load is uniformly distributed along the line of contact. In actuality, the load per unit length varies with the point of contact [3]. The finite element method is proficient to supply this information but the time required to generate proper model is large amount. CATIA5 can generate model of gear. In CATIA5 the generated model geometry is saved as a file and then transferred to ANSYS for analysis. Gear analysis can be performed using analytical methods which required a number of assumption and simplifications which aim at getting the maximum stress values only but gear analyses are multidisciplinary including calculations related to the tooth stresses. In this work, an attempt will be made to analyze bending stress to resist bending of helical gears, as both affect transmission error. Due to the progress of computer technology many researchers tended to use numerical Methods to develop theoretical models to calculate the effect of whatever is studied. Numerical methods are capable of providing more truthful solution since they require very less restrictive assumptions. However, the developed model and its solution method must be selected attentively to ensure that the results are more

acceptable and its computational time is reasonable. The dimension of the model have been arrived at by theoretical methods. The stress generated of the tooth have been analyzed for materials. Finally the results obtained by theoretical analysis, AGMA calculations and finite element analysis are compared to check the correctness. Vijayaragan and Ganesan [4] presented a static analysis of composite helical gears system using three dimensional finite element methods to study the displacements and stresses at various points on a helical gear tooth. Cheng and Tsay investigates the contact stress and bending stress of a helical gear set with localized bearing contact, by means of finite element analysis (FEA). The proposed helical gear set comprises an involute pinion and a double crowned gear. Mathematical models of the complete tooth geometry of the pinion and the gear have been derived based on the theory of gearing and a mesh-generation program was also developed for finite element stress analysis. The gear stress distribution is investigated using the commercial FEA package, ABAQUS=Standard [5]. Litvin [6] proposed the concept of tooth surface modification to obtain a pre-designed parabolic TE as well as a localized bearing contact of the gear set. This concept of tooth modification has been applied to the generation of various kinds of gearing, such as spur gears, helical gears and worm gear drives [7],[9]. The contact stress and fillet stress on gears, which are closely related to pitting failure, bending failure and the gear's service life, have attracted much attention [10],[11]. The calculation formulae for gears with special profile modifications are rarely available in handbooks [12],[13]. Therefore, Finite element analysis (FEA), which can involve complicated tooth geometry, is now a popular and powerful analysis tool to determine tooth deflections and stress distributions. Many researchers have applied FEA to tooth deflection and stress distribution for various gear drives. Several researchers have analyzed line-contact involute helical gears using three-dimensional (3-D) Finite element (FE) stress analysis [10],[11]. However, these researchers applied loads directly to the contact ellipses and contact lines obtained from tooth contact analysis (TCA). FE contact analysis for deformable bodies is complex and non-linear. Most early 3-D FE contact analyses were performed using gap elements [14]. Now, due to the progress of computer technology and computational techniques, some FEA packages can deal with contact analysis without using gap elements. Some researchers have begun to apply these FEA softwares to contact problems of gear surfaces [15],[8]. The authors have presented a generation method for the modified helical gear, possessing double crowning effects in the profile and lengthwise directions [16]. A

computer program for the FE mesh generation of a 3-D tooth model is developed from the derived tooth geometry. An FEA package, ABAQUS, capable of contact analysis for two 3-D deformable bodies was employed to determine the stress distribution of a pair of contact gear teeth in point contact [17],[18].

Bending Stress (Lewis Formula) [20]

The bending stress is one of the crucial parameters during the analysis of helical gears. When the total repetitive load acting on the gear tooth is greater than its strength then the gear tooth will fail in bending. Bending failure in gears is predicted by a formula developed by **Wiltred Lewis in 1893**. He modeled a gear tooth taking the full load at its tip as simple cantilever beam. If we substitute a gear tooth for the rectangular beam, we can find the critical point in the root fillet of the gear by inscribing a parabola.

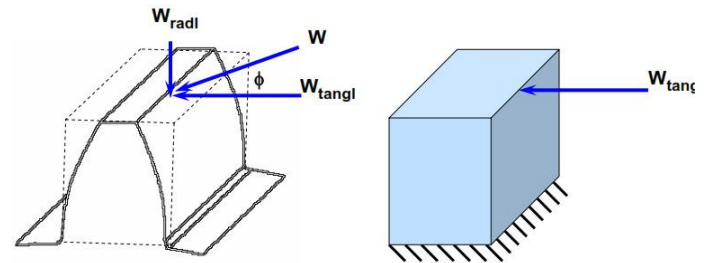


Figure 1. Tangential force

The formula uses the bending of cantilever beam to simulate the bending stress acting on the gear as shown in Figure 1, the tangential load (W_t) induces bending stress which tends to break the tooth. The maximum bending stress induced by this force Is given by,

$$\sigma_b = \frac{W_t}{F \times m \times Y} \dots\dots\dots(1.1)$$

Where, σ_b = Root bending stress (N/m²), W_t = Transmitted tangential Load (Newton), F = Face width (m or mm), m = Module (m or mm), Y = Lewis form Factor. Y is the function of number of teeth, pressure angle and an involute depth of the gear,

$$Y = \frac{2x}{3m}$$

It is fact that, when teeth mesh, the load is delivered to the teeth with some degree of impact. If we go with simply to calculate bending stress, the velocity factor is should be used in calculation. Now Lewis equation becomes,

$$\sigma_b = \frac{W_t}{K_v \times F \times m \times Y}$$

Where, K_v is the Velocity Factor.

AGMA Bending Stress Equation

Equation (1.1) is known as Lewis equation, which considers only static loading and doesn't take the dynamics of meshing teeth into account. The above stress formula must be modified to account different situations like stress concentration and geometry of the tooth. Therefore, Equation (1.2) that is shown below is the modified Lewis equation recommended by AGMA for practical gear design to account for variety of conditions that can be encountered in service.

The AGMA equation for bending stresses given by,

$$\sigma_b = \frac{F_t}{b m_n J} K_v K_o K_s (0.93 K_m)$$

.....(1.2)

Where, F_t = Normal Tangential Load

m_n = Normal Module

J = Geometry factor

K_v = Dynamic factor

K_o = Overload factor

K_s = Size factor

K_m = Load distribution factor.

Each of these factors can be obtained from machine design books [22]. This analysis considered only the component of the tangential force acting on the tooth and doesn't considered the effects of the radial force, which will cause in compressive stress over the cross section on the root of the tooth.

Hertz Contact Stress (Involute Gear Tooth Contact Stress Analysis) [22]

One of the main gear tooth failure is pitting which is a surface fatigue failure due to repetition of high contact stresses occurring in the gear tooth surface while a pair of teeth is transmitting power. Contact failure in gears is currently predicted by comparing the calculated Hertz contact stress to experimentally determined allowable values for the given material. The method of calculating gear contact stress by Hertz's Equation (2) originally derived for contact between two cylinders. In

machine design, problems frequently occurs when two members with curved surfaces are deformed when pressed against one another giving rise to an area of contact under compressive stresses. Of particular interest to the gear designer is the case where the curved surfaces are of cylindrical shape because they closely resemble gear tooth surfaces. The surface compressive stress (Hertzian stress) is found from the equation

$$\sigma_c = \sqrt{\frac{F_t}{\pi B \cos \phi} \times \frac{\frac{1}{r_1} + \frac{1}{r_2}}{\frac{1-v_1^2}{E_1} + \frac{1-v_2^2}{E_2}}}$$

Where, where r_1 and r_2 are the instantaneous values of the radii of curvature on the pinion- and gear-tooth profiles, respectively, at the point of contact. The radii of curvature of the tooth profiles at the pitch point are

$$r_1 = \frac{d_p \sin \phi}{2}, \quad r_2 = \frac{d_g \sin \phi}{2}$$

AGMA Contact Stress Equations

$$\sigma_c = C_p \sqrt{\frac{F_t \left(\frac{\cos \phi}{0.95 CR} \right) K_v K_o (0.93 K_m)}{b d I}}$$

Where, C_p is an elastic coefficient

$$\left(\sqrt{N / mm^2} \right)$$

$$C_p^2 = \left\{ \frac{1}{\frac{1-v_1^2}{E_1} + \frac{1-v_2^2}{E_2}} \right\}$$

$C_p = 190.3$ (MPa) for steel (Bhandari, 2012)

Where

$$CR = \left(\frac{\sqrt{(r_1+a)^2 - rb_1^2} + \sqrt{(r_2+a)^2 - rb_2^2} + (r_1+r_2) \sin \phi}{\pi m \cos \phi} \right)$$

$$I = \frac{\sin \varphi \cos \varphi}{2} \frac{i}{i+1}$$

The Hertz equations discussed so far can be utilized to calculate the contact stresses which prevail in case of tooth surfaces of two mating helical gears. Though an approximation, the contact aspects of such gears can be taken to be equivalent to those of cylinders having the same radii of curvature at the contact point as the load transmitting gears have. Radius of curvature changes continuously in case of an involutes curve, and it changes sharply in the vicinity of the base circle.

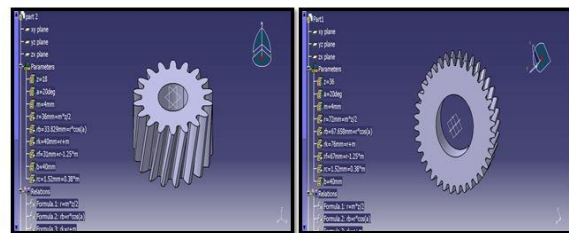
Table 1 : Dimensions of Helical Gear			
S. No.	Parameters	Pinion	Gear
1.	Number of teeth	18	36
2.	Pressure angle, normal	20°	
3.	Helix angle	15°(RH)	15°(LH)
4.	Face width (mm)	40	
5.	Normal module (mm)	4	
6.	Material	Grade 1 Steel	Grade 1 Steel
7.	Input speed (rpm) (A)	720
8.	Input power (KW) (A)	5
9.	Input speed (rpm) (B)	1500
10.	Input power (KW) (B)	35
11.	Diameter of pitch circle (mm)	72	144
12.	Diameter of base circle (mm)	67.4	135.2
13.	Diameter of Addendum circle (mm)	80	152
14.	Diameter of Dedendum circle (mm)	62.8	134
15.	Circular Pitch (mm)	12.56	
16.	Young's modulus (MPa)	2.1 × 10 ⁵	
17.	Poisson's ratio	0.3	
18.	Torque (N-m)	663.48	

Parametric Modeling of Helical Gear

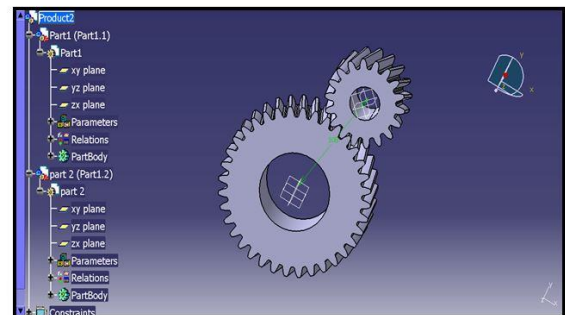
Parametric modeling allows the design engineer to let the characteristic parameters of a product drive the design of that product. During the gear design, the main parameters that would describe the designed gear such as module, pressure angle, and root radius, and tooth thickness, number of teeth could be used as the parameters to define the gear as shown in Table 1.

In this paper work, module, pressure angle, numbers of teeth of both the gears are taken as input parameters. CATIA V5 uses these parameters, in combination with its features to generate the geometry of the helical gear and all essential information to create the model. By using the relational equation in CATIA V5, the accurate three dimensional helical gear models are developed. CAD software packages allow for modeling and simulation of 3D parametric modeling of helical gear. It also a good interface with Finite Element software. CATIA has model the involute profile helical gear geometry perfectly. For helical gear in CATIA, relation and equation modeling is used. Relation is used to express dependencies among the dimension needed for defining the basic parameters on which the model is depends. The assembly of gear is done by consider the left hand Helical gear and right hand helical pinion. Then the file is saved as IGES format.

PARAMETRIC MODELING OF HELICAL GEAR AND HELICAL PINION USING CATIA V5



PINION GEAR
ASSEMBLY OF HELICAL GEAR AND HELICAL PINION



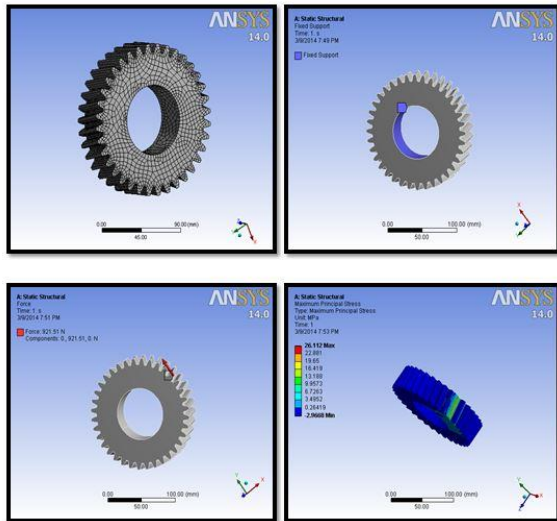
Analysis of Helical Gear

The assembly and 3D model of Helical gear which was created in CATIA V5 is imported in ANSYS workbench 14.0 for stress analysis. It is done

by saving drawing in STP or IGS file format in CATIA V5. In Ansys workbench select the static analysis from menu then connect the geometry to analysis tab. Once the geometry is loaded with static analysis tab, next is to define contact between the two involute teeth profile. The CAD Commercial software automatically reads the attached geometry for predefined contact. The teeth contact between two teeth is set as frictionless. Default setting for mesh generation is not sufficient to get accurate result. For that select proper relevance with smoothing and span angle. For boundary condition, frictionless support is given to gears as shown in Figure 4 and Torque $T = 663.48$ Nm is applied to the right helical pinion in anticlockwise direction. For finding Bending Stresses by varying Face Width Loads 921.51 N and 3096.3 N is applied to the left Helical gear.

Finite Element Analysis For Bending Stress Of Helical Gear

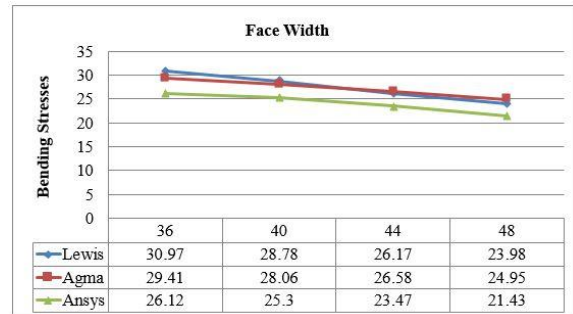
Helical gear assembly was imported in ANSYS 14.0 and the boundary conditions were applied to the gear model. The model was analyzed for the root bending stress for the applied tangential, axial and radial force. In helical gear only 3-D analysis was performed because of the helical profile of its teeth. The Figures shows the stress distribution plot along the tooth.



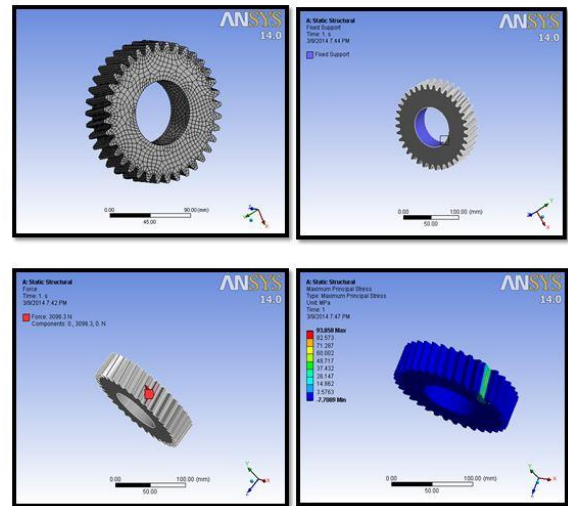
Effect of Face Width on Load 921.51 N

The effect of face width on maximum bending stress is studied by varying the face width for five different values which are ($b = 36$ mm, 40 mm, 44 mm, 48 mm). The maximum bending stresses obtained are shown in Table 2 and it is observed from the above Figure, there is a variation in the maximum bending stresses with the change in face width. The maximum bending stress value decreases with the increase of face width.

Table 2 : Comparison of Values of the Root Bending Stresses by Considering Different Face Width			
Face Width (b) (mm)	Root Bending Stresses (MPa)		
	LEWIS	AGMA	ANSYS
36	30.97	29.41	26.12
40	28.78	28.06	25.3
44	26.17	26.58	23.47
48	23.98	24.95	21.43



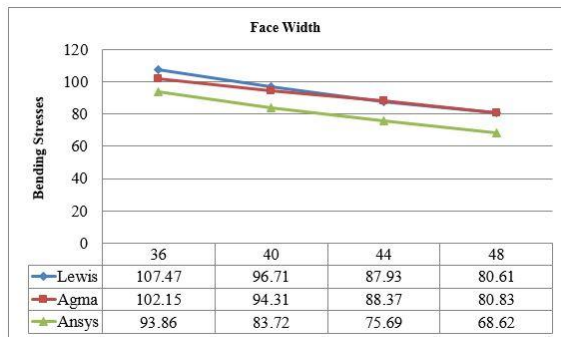
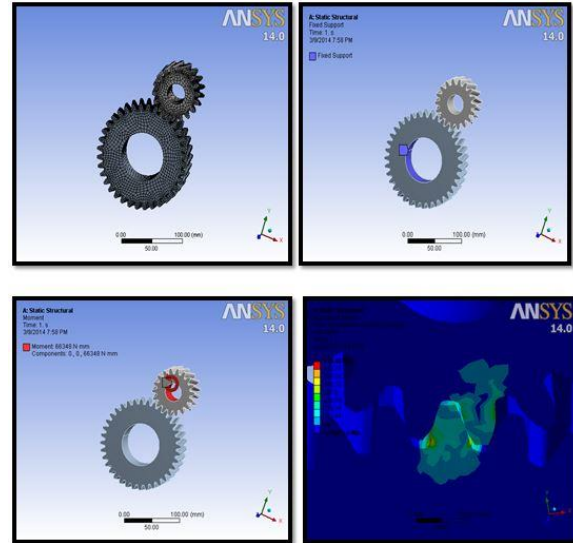
Effect of Face Width on Load 3096.3 N



The effect of face width on maximum bending stress is studied by varying the face width for five different values which are ($b = 36$ mm, 40 mm, 44 mm, 48 mm). The maximum bending stresses obtained are shown in Table 3 and it is observed from the above Figure, there is a variation in the maximum bending stresses with the change in face width. The maximum bending stress value decreases with the increase of face width.

Table 3 : Comparison of Values of the Root Bending Stresses by Considering Different Face Width

Face Width (b) (mm)	Root Bending Stresses (MPa)		
	LEWIS	AGMA	ANSYS
36	107.47	102.15	93.86
40	96.71	94.31	83.72
44	87.93	88.37	75.69
48	80.61	80.83	68.62



Comparison

In this section the modeled helical gear is analyzed to study the effect of face width on bending under static load with different parameters. Throughout the analysis each gear is studied for four different face widths ($b = 36$ mm, 40 mm, 44 mm, 48 mm). All the rest parameters and the applied load are kept constant.

FEA for Contact Stress Of Helical Gear

Contact stresses were studied in the same manner as bending stresses were calculated. In this paper Von Mises Contact stresses are obtained at the contact region.

Contact Stress

Hertz Equation	-470.64 MPa
AGMA contact stress	548.66 MPa
FEA	510.48 MPa

Results and Discussion

The structural stress analysis of the helical gear tooth model is carry out using the FEA in ANSYS 14.0. The load applied at the tooth of the helical gear by applying the analysis over the tooth which is facing the load we get the stress distribution in the numeric as well as in the form of the color scheme. By varying the face width and keeping the other parameters constant various models of the helical gear are created. For determining at any stage during the design of the gear face width is an important parameter. The results of the variation in face width from (36 mm to 48 mm) there is continuous decrement in the value of the stress of the tooth of the helical gear stress. Results of theoretical, AGMA, and ANSYS are closer, therefore the design are accepted. As it is seen clearly from all tables and graphs the maximum bending stress values are increase with the decrease of face width. The contact stress is analyzed by simulating the real contact region between the two mating gears. In this work we got on three results as follow

- Theoretical results (from Lewis equation and Hertz equation directly)
- AGMA results
- ANSYS results

And all results are closer as shown in graphs.

Conclusions

Maximum bending stress occurred in the upper half of the helical gear. In theory of helical Gear, we are considering that the load is acting at one point and the stress is calculated. The calculation of maximum stresses in a helical gear at tooth root is three dimensional problems. The accurate evaluation of stress state is complex task. The contribution of this thesis work can be summarized as follows:

- The strength of helical gear tooth is a crucial parameter to prevent failure. In this work, it is shown that the effective method to estimate the root bending stress using three dimensional model of a helical gear and to verify the accuracy of this method the results with different face width of teeth are compared with theoretical and AGMA formulas.
- In helical gear the engagement between driver gear and driven gear teeth begins with point contact and gradually extends along the tooth surface. Due to initial point contact in helical gear the bending stresses produced at critical section (root of tooth) are maximum as compared to spur gear, which has kinematic line contact.

The face width is an important geometrical parameter in design of helical gear as it is expected in this work the maximum bending stress decreases with increasing face width.

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